A Boundary Contour Mode-Matching Method for the Rigorous Analysis of Cascaded Arbitrarily Shaped H-Plane Discontinuities in Rectangular Waveguides

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***Abstract*— A rigorous boundary contour mode-matching (BCMM) method is presented for the efficient calculation of the modal scattering matrix of cascaded, arbitrarily shaped H-plane discontinuities, junctions and/or obstacles in rectangular waveguides. For the inhomogeneous waveguide region with general contour, the field is expanded in the complete set of cylindrical wave functions. At the boundary to the ports with homogeneous waveguide sections, the mode-matching technique is applied that yields the modal scattering matrix of the corresponding key-building block directly. To show the usefulness of the method, the filter design of inductive iris coupled resonators with rounded corners is presented. The theory is verified by comparison with results obtained by other methods.**

I. Introduction

RECTANGULAR waveguide H-plane discontinuities play an important role in the design of many microwave components, such as filters [1]–[5], tapers [3], phase shifters [6], and circulator elements [7]. Although rectangular [2], [5] and circular structures [1], [4], [5] are commonly used, there is increasing interest in discontinuities of more general shape which offer the advantage that modern fabrication methods may be easily applied, like computer controlled milling or spark eroding techniques. Moreover, the additional degrees of freedom yield the potential of supplementary design parameters.

Several techniques have already been applied for analyzing the scattering behavior of waveguide H-plane discontinuities. For structures of arbitrary shape, the moment method [1], [4], [8], the boundary element method [3], the finite element method [9], the combined finite and boundary element method [6], and a combined modal expansion point matching technique [7] have been reported so far.

The purpose of this letter is to present a new and general field-theory method, the boundary contour mode-matching (BCMM) method, for the efficient and rigorous calculation of the modal scattering matrix of cascaded, arbitrarily shaped H-plane discontinuities, junctions and/or obstacles in rectangular waveguides (Fig. 1). In the inhomogeneous waveguide region of the single key-building block discontinuity of arbitrary contour (Fig. 1), the field is expanded in the complete set of cylindrical wave functions, similar to the H-plane ferrite post problem in [7] and to the cross-section analysis problem in [10]. However, instead of the point matching used in [7] the mode-matching technique at the boundary to the waveguide ports is applied that yields quick convergence and high numerical stability. Moreover, the modal scattering matrix of the corresponding region is obtained directly. The generalized modal scattering matrix technique achieves the rigorous and flexible modeling of composed H-plane waveguide structures of arbitrary cross-section, including the higher-order mode interactions at all discontinuities, such as of inductive iris coupled filters with resonators of more general shape (Fig. 4). The theory is verified by comparison with results obtained by other methods.

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II. Theory

For the single key-building block discontinuity (Fig. 1), the fields

$$\vec{E}^{\nu} = \frac{1}{j\omega\epsilon}\nabla \times \nabla \times \left(\vec{A}^{e\nu}\right) + \nabla \times \left(\vec{A}^{m\nu}\right)$$
$$\vec{H}^{\nu} = \frac{-1}{j\omega\epsilon}\nabla \times \nabla \times \left(\vec{A}^{m\nu}\right) + \nabla \times \left(\vec{A}^{e\nu}\right) \qquad (1)$$

in the subregions $\nu =$ I, II (i.e., in the homogeneous waveguides) are derived from the z-component of the magnetic vector potential

$$A_z^{m\nu} = \sum_{m=1}^{M} N_m^{\nu} \cos\left(m\pi\frac{x_\nu}{a_\nu}\right) \times \left[a_m^{\nu} e^{\mp\gamma_m^{\nu}(z-z_r^{\nu})} + b_m^{\nu} e^{\pm\gamma_m^{\nu}(z-z_r^{\nu})}\right], \qquad (2)$$

which is expressed (TE_{m0} incidence assumed) in terms of the corresponding waveguide eigenmodes; N_m^{ν} is a normalization factor [2], [11], with regard to the complex power carried by each mode, to yield directly the corresponding modal scattering matrix at the boundary z_r^{ν}; a_m and b_m are the still unknown eigenmode amplitude coefficients.

In the subregion $\nu =$ III (i.e., the inhomogeneous waveguide region with arbitrary contours Γ_1, Γ_2) which is assumed to be uniform in y, the fields (1) are derived from the y-component of the electric vector potential. This is expanded

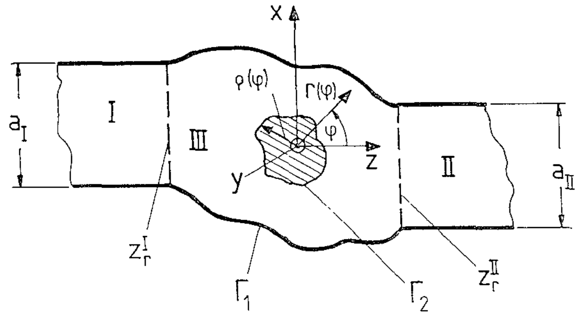

Fig. 1. Single key-building block H-plane discontinuity of arbitrary contour.

in terms of the complete set of cylindrical wave functions (i.e., the solutions of the corresponding general wave equation)

$$A_y^{e\nu} = \sum_{n=0}^{N} B_n(r(\varphi))[c_n \cos(n\varphi) + d_n \sin(n\varphi)] \cdot \cos\left(p\pi \frac{y}{b}\right). \quad (3)$$

B_n is a combination of Bessel functions of the first and second kind, J_n and N_n, respectively

$$B_n = f_n J_n(k_{rp} r(\varphi)) + g_n N_n(k_{rp} r(\varphi)), \quad (4a)$$

which reduces to

$$B_n = J_n(k_{rp} r(\varphi)) N_n(k_{rp}\rho_0) - J_n(k_{rp}\rho_0) N_n(k_{rp} r(\varphi)), \quad (4b)$$

for the special case of a circular boundary Γ_2 with the radius ρ_0, and to

$$B_n = J_n(k_{rp} r(\varphi)), \quad (4c)$$

for the special case of an empty subregion III ($\rho = 0$). k_{rp} is the wavenumber in r direction that is related to the free space wavenumber k by

$$k_{rp}^2 = k^2 - \left(\frac{p\pi}{b}\right)^2; \quad (5)$$

b is the uniform waveguide height, and for TE_{m0} incidence, $p = 0$. The expansion coefficients c_n, d_n, f_n, and g_n in (3) depend on the shape of the boundary contours Γ_1 and Γ_2 (Fig. 1) as well as on the modal field distribution over the cross-section and on the port regions. Note that the boundary Γ_1 consists of the metallic contour of the waveguide sidewall, where the tangential electric field vanishes, and of the arbitrarily chosen boundaries z_r^ν to the waveguide ports, where the tangential electric field is finite and has to be matched to the corresponding field in the waveguide section.

A Fourier expansion of the tangential electric field on the contours Γ_1 and Γ_2 in region $\nu = \mathrm{III}$

$$E_y(\varphi) = \sum_{i=0}^{\mathrm{I}} \alpha_i \cos(i\varphi) + \beta_i \sin(i\varphi), \quad (6)$$

satisfies the given field periodicity with respect to the angular coordinate φ. The Fourier coefficients, α_i, β_i in (6) are related to the still unknown coefficients c_n, d_n by (1) and (3) in the form (where for simplicity cases (4b) or (4c) are assumed)

$$\begin{bmatrix} (\alpha) \\ (\beta) \end{bmatrix} = \begin{bmatrix} (CE_{CC}) & (CE_{CS}) \\ (CE_{SC}) & (CE_{SS}) \end{bmatrix} \begin{bmatrix} (c) \\ (d) \end{bmatrix}. \quad (7)$$

The coupling integrals in (7) are given by

$$\begin{bmatrix} (CE_{CC}) & (CE_{CS}) \\ (CE_{SC}) & (CE_{SS}) \end{bmatrix} = \int_{(2\pi)} B_n(r(\varphi)) \cdot \begin{bmatrix} \cos(i\varphi) \\ \sin(i\varphi) \end{bmatrix} \cdot \begin{bmatrix} \cos(n\varphi) & \sin(n\varphi) \\ \cos(n\varphi) & \sin(n\varphi) \end{bmatrix} d\varphi. \quad (8)$$

The Fourier expansion of the tangential electric field in regions $\nu = \mathrm{I}, \mathrm{II}$ relates the corresponding Fourier coefficients α^{wg}, β^{wg} via (1), (2) to the still unknown eigenmode amplitude coefficients a_m and b_m of (1) in the form

$$\begin{bmatrix} (\alpha^{wg}) \\ (\beta^{wg}) \end{bmatrix} = \sum_{l=1}^{L_0} \begin{bmatrix} (AE_c^l) \\ (AE_s^l) \end{bmatrix} (a^l) + \begin{bmatrix} (BE_c^l) \\ (BE_s^l) \end{bmatrix} (b^l), \quad (9)$$

where L_0 denotes the number of waveguide ports.

By matching the tangential field components at the boundaries z_r^{I} and z_r^{II}, the still unknown expansion coefficients c_n, d_n of (3) may be immediately expressed in terms of the eigenmode amplitude expansion coefficients a_m, b_m of the waveguide regions I and II, respectively. Note that for the tangential magnetic field along the contours Γ_1, Γ_2 (which is defined only for the port cross-sections) the standard mode matching may be applied. Rearranging of the equations yields the modal scattering matrix of the corresponding key-building block discontinuity, directly. The modal scattering matrix of cascaded structures is calculated by the known generalized S-matrix technique [2].

III. Results

Fig. 2 demonstrates the convergence behavior of the BCMM method at the example of a rectangular H-plane 90°-bend in the waveguide Ku-band (WR 62 waveguide: 15.799 mm × 7.899 mm). M denotes the number of waveguide modes considered in the homogeneous port waveguides (I, II), N is the number of cavity eigenmodes (III). Good convergence behavior may be stated as compared with the result of the standard mode-matching method [11] obtained by 15 eigenmodes.

Fig. 3 shows the calculated insertion loss curve of a two-element circular post filter according to [1]. Good agreement with the results of the moment method in [1] may be observed.

In Fig. 4, the design results are presented of a Ku-band (WR 62 waveguide) broad-band filter with inductive iris coupled resonators with rounded corners. Such filters may be of interest for modern fabrication methods, like computer controlled milling or spark eroding techniques, where often corners with finite radii are realistic. As is demonstrated in Fig. 4, the radius R_0 is of perceptible influence on the midband frequency and the return loss ripple, already for relatively small values of R_0.

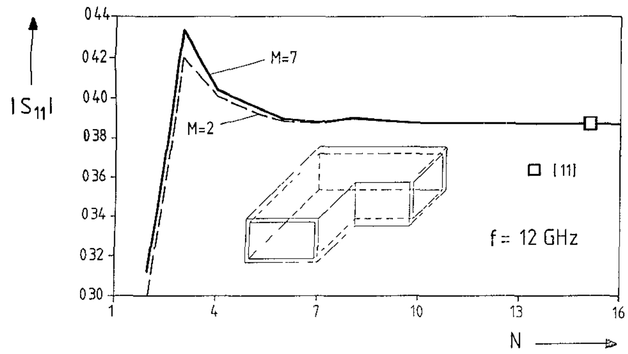

Fig. 2. Convergence behavior of the BCMM method at the example of a rectangular H-plane 90°-bend in the waveguide Ku-band (WR 62 waveguide: 15.799 mm × 7.899 mm). (M denotes the number of waveguide modes considered in the homogeneous port waveguides, N is the number of cavity eigenmodes.

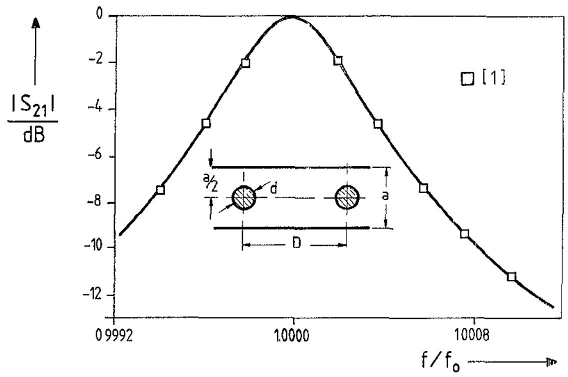

Fig. 3 Insertion loss of a two-element circular post filter according to [1]: $\lambda_0/a = 1.2$, $D/\lambda_{g0} = 0.62522$, $d/a = 0.3$.

IV. Conclusion

The rigorous boundary contour mode-matching (BCMM) method presented in this letter achieves the efficient, flexible and accurate design of cascaded, arbitrarily shaped H-plane discontinuities, junctions and/or obstacles in rectangular waveguides. Since the theory includes the finite thickness and general shape of the structures as well as the higher-order mode interaction of all discontinuities, all relevant design parameters may be rigorously taken into account in the optimization process. The design method is well compatible with modern fabrication methods, like computer controlled milling or spark eroding techniques, where often structures of more general shape, e.g., rounded corners, have to be taken into account.

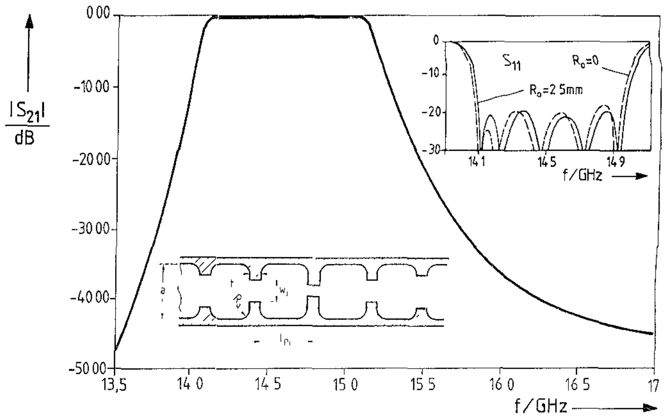

Fig. 4 Computer optimized design results of a five–resonator Ku–band (WR 62 waveguide) broad-band filter with inductive iris coupled resonators with rounded corners. Filter dimensions (in mm): $a = 15.799$, $b = 7.899$, $t = 5$, $R_0 = 2.5$, $w_1 = w_6 = 10.276$, $l_1 = l_5 = 8.029$, $w_2 = w_5 = 8.291$, $l_2 = l_4 = 9.887$, $w_3 = w_4 = 7.822$, $l_3 = 10.188$.

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